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TWO SHORT METHODS FOR COMPUTING THE ORBIT OF A SPECTROSCOPIC BINARY STAR BY USING THE ALLEGHENY TABLES OF ANOMALIES.

BY F. HENROTEAU.

Several methods have been proposed for deriving the elements of spectroscopic binary stars, but they involve either a certain amount of computation or a certain amount of graphical construction, or else the use of curves that have been drawn once for all on tracing cloth, as in the elegant method due to King.

Since the Allegheny Tables are used to pass from the elements to the computation of an ephemeris, it would seem that they might also be used for the reverse operation, that is, to pass from the radial velocities to the elements of the orbit. Two methods have been devised, following that idea, of which the second one especially is very expeditious and not only allows us to get preliminary elements but affords a criterion to test them and in a very short time, by successive approximations, to secure very good elements. The two methods are given here at length with their theoretical explanations. The second one has been used for several orbits of different eccentricities and has always given good results.

FIRST METHOD, FOR FINDING THE POINT CORRESPONDING TO PERIASTRON AND THE ECCENTRICITY.

Suppose that the velocity curve has been determined by a series of observations. We know that the equation of the velocity curve is, if the mean axis XX_2 is taken as axis of abscissae,

$V = K \cos u$, (1)

where V is the velocity and $u = v + \omega$, v being the true anomaly, ω the angular distance of periastron from the ascending node, K the semi-amplitude *measured* on the velocity curve *with the same unit-length as the times or abscissae* (K is also the absolute value of the ordinates of maximum or minimum, since XX_2 is the mean axis).

From (1) we obtain by differentiating:

$$\frac{dV}{dt} = -K \sin u \frac{dv}{dt}; \quad (2)$$

from which we have

$$\frac{dv}{dt} = -\frac{dV}{dt} \times \frac{1}{K \sin u} \quad . (3)$$

$\frac{dV}{dt}$ is the slope of the tangent at any point A of the velocity curve.

It is also clear that if we first multiply $\frac{dv}{dt}$ by $\frac{180}{\pi}$ to transform v into degrees, and secondly multiply dt by μ or $\frac{360}{U}$, the mean daily motion (U being the period expressed in days), $\frac{dv}{dt}$ will become the slope of the tangent at a point corresponding to A of the curve having for abscissae the mean anomalies and for ordinates the true anomalies of the orbit (We shall call this the anomaly curve). Suppose P is the point corresponding to the periastron on the velocity curve. Let us call P' its projection on XX_2 and let A' be the projection of A. $P'A'$ is the time required by the star to go from periastron to the position corresponding to A. So if we know P, $\mu \times P'A'$ will be the abscissa of the point corresponding to A in the anomaly curve.

If we now take the tables for the true anomaly in elliptic orbits published by Dr. Schlesinger and Miss Udick in Volume 2 of the Allegheny Observatory Publications, for each value of the eccentricity we have the abscissae M (mean anomalies), the ordinates v, and the *slopes of the tangents* Δ corresponding to each value of M. (In other words, Δ, the difference between two successive true anomalies, differs so little from the slope of the tangent that it may be taken as such.)

Let X, X_1 and X_2 be the points where the velocity curve crosses the mean axis. If $XX_1 < X_1X_2$, P' will be on XX_1 and if $XX_1 > X_1X_2$, P' will be on X_1X_2. We may suppose P' between X and X_1, the following reasoning being the same if it should be between X_1 and X_2.

Let $+\xi$ and $-\eta$ be the slopes of the tangents to the velocity curve at X and X_1. We have from (3) and since u at these points equals 270° and 90°

$$\text{at } X, \frac{dv}{dt} = \frac{\xi}{K} \text{ and at } X_1 \frac{dv}{dt} = \frac{-\eta}{K}$$

from which the corresponding slopes of the tangents to the anomaly curve can be easily determined by multiplying by $\frac{U}{2\pi}$ as has been shown above. Let $-\Delta_1$ and Δ_2 be these slopes.

If P' should be known then $-\mu \,.\, P'X = -M_x$ and $\mu \,.\, P'X_1 = M_y$ would be the ordinates of the points corresponding to X and X_1 on the anomaly curve (if, as is understood, the point corresponding to periastron is the origin in this curve).

We do not know yet M_x nor M_y, but if we call $\mu \,.\, XX_1 = M_s$ we know that $M_x + M_y = M_s$. Therefore, if having computed Δ_1 and Δ_2 we look in which column of the Allegheny Tables they are situated so that the sum of their corresponding mean anomalies $M_x + M_y = M_s$ (M_s being deduced easily from the velocity curve), we shall read at the top of the column the eccentricity, and, moreover, the value of M_x that we shall read from the table will give us the position of the periastron. It will be quite unnecessary for obtaining a preliminary orbit, to estimate the fractions of degrees for the values of M_s, M_x and M_y.

The accuracy of the present method depends solely on the accuracy with which the tangents to the velocity curve at X and X_1 are drawn. A little difference in the way of drawing them may sometimes make great differences in the values of their slopes. A very important criterion will show us if the tangents have been accurately drawn, and this is that $v_x + v_y = 180°$ if we call $-v_x$ the true anomaly at X and v_y the true anomaly at X_1. We have indeed at X $\cos(-v_x + \omega) = 0$ and at X_1, $\cos(v_y + \omega) = 0$ and therefore $v_y + \omega = 270°$ and $-v_x + \omega = 90°$, so that by subtracting, $v_x + v_y = 180°$. If we do not find $v_x + v_y = 180°$ we shall have to draw the tangents more accurately until the criterion is satisfied.

Example of the Computation of the Eccentricity and Determination of the Position of Periastron.

From a velocity curve

$$\xi = +2.01 \quad \eta = +1.16 \quad K = \frac{41}{25} \quad XX_1 = 2^d.80 \quad U = 5^d.97.$$

Compute the eccentricity and the position of the periastron.

We have

$$\Delta_1 = \frac{2.01 \times 25 \times 5.97}{41 \times 2 \times 3.1416} = 1.16$$

$$\Delta_2 = \frac{1.16 \times 25 \times 5.97}{41 \times 2 \times 3.1416} = 0.67$$

$$M_x + M_y = M_s = \frac{360 \times 2.8}{5.97} = 169.$$

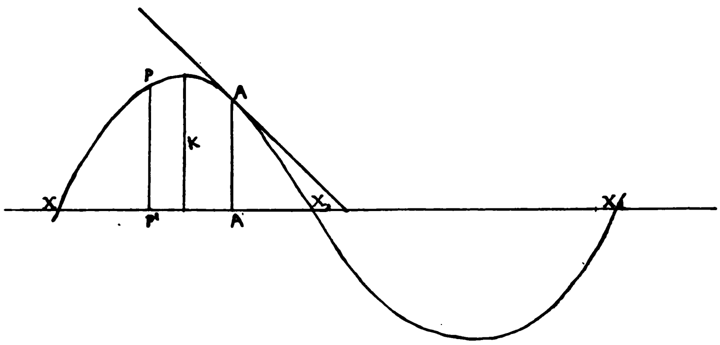
From the Tables we obtain

$$\begin{array}{ll} e = 0.31 & \\ M_x = 55^\circ & v_x = 89^\circ.9 \\ M_y = 114^\circ & v_y = 141.0 \\ \hline & v_x + v_y = 230^\circ.9 \end{array}$$

The criterion is not satisfied, so we have to conclude that the values of ξ and η have not been accurately determined. The tangents at the curve will have to be redrawn. A little practice in the use of the method will soon give us very good tangents.

Second Method For Finding the Eccentricity.

Altho the preceding method is good it is not advisable to use it if the position of the periastron point can be determined without great error by Schwarzschild's method, for in this second method the use of a criterion will permit us to correct the position of that point and obtain very rapidly an accurate position. Schwarzschild's method consists mainly in this: Lay a piece of semi-transparent paper over the velocity curve, and copy on this the curve and the mean axis, marking also the points o, $\frac{U}{2}$, U and $\frac{3U}{2}$. Shifting this copy bodily along the mean axis for a distance $\frac{U}{2}$ or half the period, rotate the copy 180° about the mean axis—i. e., turn the copy face downward on the original curve, keeping the mean axis in coincidence, and bring the point o or U of the copy over the point $\frac{U}{2}$ of the curve. The curves will then cut one another in general in four points of which two will be the points of periastron and apastron. The proper two points will be found without difficulty, for peri- and apastron must be separated in time by one half a revolution, and must moreover, lie on different branches of the velocity curve. To determine which is periastron we have the criterion, that the curve is for a shorter time on that side of the mean axis on which the point of periastron lies.



Now, when the periastron P is determined (see figure) we have $P'X \cdot \frac{360}{U} = M_x$ and $P'X_1 \cdot \frac{360}{U} = M_y$. We then take M_x and M_y in the Allegheny Tables and find out in which column the corresponding v_x and v_y have a sum $= 180°$. The eccentricity read at the top of the column will be the eccentricity of the orbit. Moreover having v_y we have $\omega = 270° - v_y$.

Then, to finish, the equations

$$\gamma = \gamma' - Ke \cos \omega = \gamma' - (e \times PP')$$

$$a \sin i = [4.13833]\ KU \sqrt{1 - e^2}$$

in which the constant of attraction is given by its logarithm and U is the period expressed in days, will give us the velocity of the center of mass and the product of the semi-major axis of the true orbit by the sine of the inclination of the orbit.

This last method of finding the elements of a spectroscopic binary star is by far the simplest of any yet known. It requires a very small amount of computation. It is mainly a reading off the tables.

For this second method we have also a very important criterion. If P is really the periastron point after we have $\omega = 270° - v_y$, we must have $\cos \omega = \frac{PP'}{K}$. If this equation is not satisfied, P is not the periastron point, and a better position of this point must be found. By the use of this criterion, even without using Schwarzschild's method at all, and only assuming first for P an arbitrary position, we may by a series of approximations find its true position. For instance, having computed $\omega = 270° - v_y$ for the arbitrary position we shall have $PP' = K \cos \omega$; a better position of P will then be that having for ordinate $PP' = K \cos \omega$; this new position of P will then give us a new $\omega = 270° - v_y$ and again a new P until the true position has been found. The use of Schwarzschild's method to obtain a first position of P is however advisable.

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